B

Illowsky – Chapt. 3 & 4

Larson – Chapt. 3 & 4

Solution

Math 123: MT 2 Formula Sheet

Probability

$$P(E) + P(E') = 1$$

$$P(A \text{ and } B) = P(A).P(B) \quad \textit{if independent}$$

$$P(A \text{ and } B) = P(A).P(B|A) \quad \textit{if dependent}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \textit{if A and B are NOT mutually exclusive}$$

$$P(A \text{ or } B) = P(A) + P(B) \quad \textit{if A and B are mutually exclusive}$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

Discrete Probability Distribution: $\mu = \sum x P(x)$, $\sigma^2 = \sum (x - \mu)^2 P(x)$,

$\sigma = \sqrt{\sigma^2} = \sqrt{\sum (x - \mu)^2 P(x)}$, Expected Value $= E(x) = \mu = \sum x P(x)$

Binomial Distribution: $P(x) = {}_nC_x\, p^x q^{n-x}$, $\mu = np$, $\sigma^2 = npq$, $\sigma = \sqrt{\sigma^2} = \sqrt{npq}$

Geometric distribution: $P(x) = p\,q^{x-1}$

Poisson distribution: $P(x) = \frac{\mu^x e^{-\mu}}{x!}$, *where* $e \approx 2.718$

Math 123, Fall 16, Midterm 2 **Name:** Solution

Instructor: Saba Gerami **Total: 100 Points**

Directions:

- Show all your work.
- You only receive half of the points if you do not explain your reasoning.
- You can use a non-graphing calculator.
- You may not use cell phone, or notes.

1. Determine whether the events are mutually exclusive or not. Explain your reasoning.

(4 points)

a) Event A: Randomly select a red jelly bean from a jar.
Event B: Randomly select a yellow jelly from the same jar.

mutually exclusive bc they cannot happen @ the same time.

b) Event A: Randomly select a person who loves cats.
Event B: Randomly select a person who loves dogs.

not mutually exclusive bc they can happen at the same time (possible to love cats & dogs)

2. Determine whether the events are independent or dependent. Explain your reasoning.

(4 points)

a) Eating all of your cousin's candies after Halloween and getting a high blood sugar after.

dependent bc eating candy will result in high blood sugar

b) Rolling a fair 5-sided die twice and getting a one both times.

independant bc the first one won't result in getting another one.

3. Perform the indicated calculations without using a calculator. Write down all the steps.

(4 points)

a) ${}_{13}P_6 = \frac{13!}{(13-6)!} = \frac{13!}{7!} = \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}} = 1,235,520$

b) ${}_{6}C_5 = \frac{6!}{5!\ 1!} = \frac{6 \times \cancel{5!}}{\cancel{5!}} = 6$

2

4. answers may vary! *(4 points)*

a) Write an event that has probability of 1.

grabbing a yellow ball from a jar full of yellow balls only.

b) Write an event that has probability of 0.

grabbing a black ball from a jar full of yellow balls only.

5. Determine whether the distribution is a probability distribution. Explain. *(4 points)*

x	20	30	40	50	60
P(x)	0.08	0.13	0.19	0.07	0.05

① all P(x) are between 0 & 1 ✓

② $\sum P(x) = 0.08 + 0.13 + 0.19 + 0.07 + 0.05$

$= 0.52 \neq 1$ X

Not a prob. distribution.

6. You are given that $P(A) = 0.5$ and $P(B) = 0.27$. Do you have enough information to find $P(A \text{ or } B)$? Explain your reasoning. *(2 points)*

No! Bc we don't know if they are mutually exclusive or not so we don't know which formula to pick.

7. AHC conducted a survey to determine if students believe that they are ready for adult life. Here are the responses: *(4 points)*

Response	Prepared	Somewhat prepared	Slightly prepared	Not prepared	Not sure
Number of times, f	259	952	552	337	63

$= \sum f = n = 2163$

a) Find the probability that a randomly selected student believes he/she is prepared.

$P(\text{prepared}) = \dfrac{259}{2163} = 0.12$

b) Find the probability that a randomly selected student is somewhat or slightly prepared .

$P(\text{somewhat prep. or slightly prep.}) = \dfrac{952+552}{2163} = 0.70$

mut. exclusive

/14

8. A probability experiment consists of rolling an 8-sided die. Find the probability of the event:

(8 points)

a) Rolling a number less than 6 $= 1, 2, 3, 4, 5$

2 $P(x<6) = \frac{5}{8} = 0.625$

b) Rolling an odd number $= 1, 3, 5, 7$

2 $P(\text{odd}) = \frac{4}{8} = \frac{1}{2} = 0.5$

c) Rolling an even number OR a number less than 3

4 $P(\text{even OR less than 3}) = P(\text{even}) + P(\text{less than 3}) - P(\text{both})$

2 4 6 8   1 2   2

$= \frac{4}{8} + \frac{2}{8} - \frac{1}{8} = \frac{5}{8} = 0.625$

not mut. exclusive

9. In a jury selection pool, 65% of the people are female. Of these 65%, one out of four works in a health field. However, only 12% of men in the selection pool are in a health field.

(4 points)

a) Find the probability that a randomly selected person from the jury pool is female and works in the health field.

2

0.65 ♀ — 0.25 health; 0.75 not health

0.35 ♂ — 0.12 health; 0.88 not health

$P(\text{female \& health}) \overset{\text{dep}}{=} P(f) \times P(\text{health} \mid f)$

$= 0.65 \times 0.25 = 0.1625$

b) Find the probability that a randomly selected person from the jury pool is male and works in the health field.

2 $P(\text{male \& health}) \overset{\text{dep}}{=} P(\text{male}) \times P(\text{health} \mid \text{male})$

$= 0.35 \times 0.12 = 0.042$

2

10. We are picking two cards from a deck of time one by one and replacing it each time before picking the next one. *(6 points)*

a) Find the probability that we get a king on the first draw AND a queen on the second draw.

3 $$P(\text{King And queen}) \overset{\text{indep}}{=} P(K) \times P(Q) = \frac{4}{52} \times \frac{4}{52} = 0.005 \text{ or } \approx 0.006$$

b) Find the probability that we get a king on the first draw OR a queen on the second draw.

3 $$P(\text{king or queen}) \overset{\text{mut. exc}}{=} P(K) + P(Q) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.15$$

11. Ten adults enter an ice-cream-eating race. How many ways can the ice-cream eaters finish first, second, and third? *(4 points)*

order matter $\Rightarrow$ $${}_{10}P_3 = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}} = 720$$

2 — or by calculator — 2

12. AHC wants to send 5 people to the ice-cream-eating contest. Twelve people are interested in going. How many different groups can we send to the contest? *(4 points)*

$${}_{12}C_5 = \frac{12!}{5!\,7!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times \cancel{7!}}{1 \times 2 \times 3 \times 4 \times 5 \times \cancel{7!}} = 792$$

$\Rightarrow$ order does not matter — 2 — or by calculator — 2

/14

13. Chris has 11 trees that he wants to plant in a row. *(8 points)*

a) How many different ways can he plant all of them if each one is different?

2pts $11!$ or ${}_{11}P_{11}$ or $39,916,800$

b) He is changing his mind! He only wants to plan 5 of these trees so he gets rid of 6 of them. How many different ways can he plant the 5 trees that he kept?

3pts ${}_{11}P_5 = \frac{11!}{6!} = 55,440$

or by calculator

c) Chris is changing his mind again! What a mess.. This time he decided he wants to plant 11 trees again: 5 maple trees, 3 oaks, and 3 cedars. How many different ways can he do that?

3pts $\frac{11!}{5!\,3!\,3!} = 9240$

14. Find the probability of being dealt three hearts from a standard deck of cards. *(5 points)*

${}_{13}C_3$ ${}_{52}C_3$

$P(3 \text{ hearts}) = \frac{{}_{13}C_3}{{}_{52}C_3} = \frac{11}{850} \approx 0.0129$

≈ 0.013

5pts

13

15. You are invited to a fancy wedding where you can choose one of 4 available salad, one of 5 available manin courses, and 2 of 10 available desserts. How many different ways can you eat that night if you wish to have a salad, a main course and two desserts. *(5 points)*

5pts

$$\underbrace{4}_{\text{salad}} \times \underbrace{5}_{\text{main course}} \times \underbrace{{}_{10}C_2}_{\text{dessert}} = 900$$

16. Find the mean, variance, and standard deviation of the discrete probability distribution <u>using columns as shown in class.</u> *(8 points)*

x	0	1	2	3	4
Probability	0.16	0.22	0.28	0.20	0.14

X	P(x)	X P(x)	$X - \overset{1.94}{\overset{\shortparallel}{\mu}}$	$(X-\mu)^2$	$(X-\mu)^2 \cdot P(x)$
0	0.16	0	−1.94	3.7636	0.60
1	0.22	0.22	−0.94	0.8836	0.19
2	0.28	0.56	+0.06	0.0036	0.001
3	0.20	0.60	+1.06	1.1236	0.22
4	0.14	0.56	+2.06	4.2436	0.59
		$\Sigma x P(x) = 1.94$			$\Sigma = 1.60$

$\mu = 1.94$

$\sigma^2 \rightsquigarrow \sigma = \sqrt{1.6} = 1.2$

Mean: 1.94 $= \mu$ variance: 1.60 $= \sigma^2$ standard deviation: 1.27

/13

17. A fundraising lottery sells 1000 tickets at $2 each. There is one $200 prize, two $100 prizes and four $50 prizes. Let x= your possible winnings if you buy a single ticket. Find the expected winnings, and use this value to explain mathematically why you should or should not play this lottery. *(5 points)*

	X	P(X)	X P(X)
Win 200	198	$\frac{1}{1000}$	0.198
win 100	98	$\frac{2}{1000}$	0.196
Win 50	48	$\frac{4}{1000}$	0.192
no win	-2	$\frac{993}{1000}$	-1.99
			Σ

$\Sigma x P(x) = -1.4$

on average you lose $1.404.

do not play!

18. About 12% of Santa Maria drivers don't wear seat belts. *(5 points)*

a) Identify *Success*, *Fail*, p and q.

S = don't wear seat belt $\rightarrow P = 0.12$

F = wear seat belt $\rightarrow q = 0.88$

1-p

b) Find the mean, variance, and standard deviation of the number of Santa Maria drivers who do not wear seat belts if we are randomly picking 100 drivers from Santa Maria.

n= 100

Binomial

mean $= np = 100(0.12) = 12$

variance $= \sigma^2 = npq = (100)(0.12)(0.88) = 10.56$

Std. dev $= \sigma = \sqrt{\sigma^2} = \sqrt{10.56} = 3.25$

10

$P = 0.3$

$q = 0.7$

n

8

Binomial

19. About 30% of U.S. adults are trying to eat healthier. You randomly select eight adults. *(~~6~~ points)*

a) Find the probability that the number of adults who say they are trying to eat healthier is exactly 3.

$$P(3) = {}_8C_3 \, (0.3)^3 (0.7)^5 = 0.25$$

b) Find the probability that the number of adults who say they are trying to eat healthier is less than 2.

add bc mut. exclusive

$$P(x<2) = P(x=0 \text{ or } x=1) \stackrel{\downarrow}{=} P(0) + P(1)$$

$$= {}_8C_0 (0.3)^0 (0.7)^8 + {}_8C_1 (0.3)^1 (0.7)^7 = 0.058 + 0.198$$

$$= 0.256$$

20. Twenty-two percent of former smokers say they tried to quit four or more times before they were habit-free. You randomly select 12 former smokers. Find the probability that the first person who tried to quit four or more times is the third person selected. *(3 points)*

n $\quad$ x $\leftarrow$ Geometric

$P = 0.22$

$q = 0.78$

$$P(x=3) = (0.22)(0.78)^{3-1} = (0.22)(0.78)^2 = 0.133$$

Poisson

21. During a 12-year period, sharks killed an average of 5 people each year worldwide. Find the probability that the number of people killed by sharks next year is exactly 4. *(3 points)*

$x = 4$

$$\mu = 5$$

$$P(x=4) = \frac{5^4 \cdot (2.718)^{-5}}{4!} = 0.176$$

$8/12$