A

Illowsky – Chapt. 6, 7, & 8
Larson – Chapt. 5 & 6

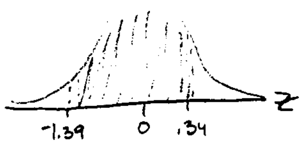
excellent! 100

Math 123 Exam 3 SHOW ALL WORK Name

1. Find each of the following probabilities. Illustrate each answer with a sketch.

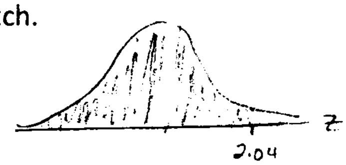
a. $P(-1.39 < Z < 0.34)$

$\boxed{0.5508}$



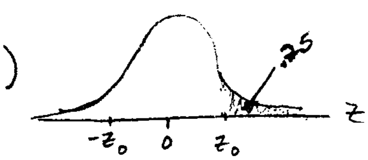
b. $P(Z < 2.04)$

$\boxed{0.9793}$



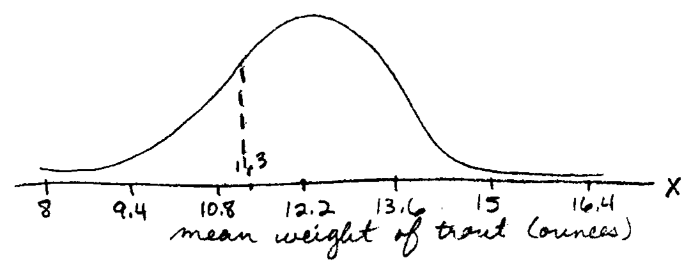
2. Find the value Zo such that 25% of the area under the standard normal curve lies to the right of Zo. Illustrate your answer with a sketch.

$\boxed{z_0 = .67}$ invnorm (.75)



3. Suppose that the weights of trout in a stream are normally distributed with a population mean of 12.2 ounces and a standard deviation of 1.4 ounces.

a. Make a sketch of this distribution. Be sure to label the horizontal axis clearly, including tick marks for each standard deviation.



b. You catch a trout at random from this stream. What is the probability that the trout's weight will exceed 11.3 ounces?

$P(x > 11.3)$ $\quad z = \frac{11.3 - 12.2}{1.4} = -0.643$

$P(z > -0.643)$

$\approx \boxed{0.7399}$

4. Suppose that the weights of trout in a stream are normally distributed with a population mean of 12.2 ounces and a standard deviation of 1.4 ounces. You take a random sample of 17 trout from this stream.

$17 < 30$ but normal

a. What values would you predict for the mean and standard deviation of the sampling distribution of the sample mean?

$$\mu_{\bar{x}} = \boxed{12.2} \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.4}{\sqrt{17}} \approx \boxed{0.3395}$$

b. What is the probability that the mean weight from your sample is between 10.3 and 11.4 ounces? $P(10.3 < \bar{x} < 11.4)$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$z = \frac{10.3 - 12.2}{0.3395} = -5.60$$

$$z = \frac{11.4 - 12.2}{0.3395} = -2.36$$

$$P(-5.60 < z < -2.36) = \boxed{0.0091}$$

c. Could you answer part b if you did not know that trout weights were normally distributed? Explain very clearly. $17 < 30$

No; because the sample size would need to be ≥ 30 if we did not know that the weights were normally distributed.

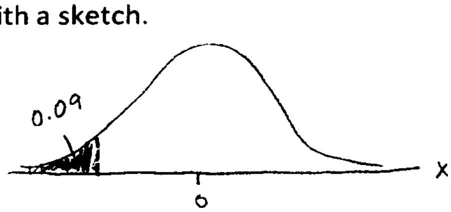
5. In the stream described in problems 3 and 4, what weight would be the cutoff for the lightest 9% of trout in the stream? Illustrate your answer with a sketch.

$z = \text{invnorm}(.09) \approx -1.34$

$x = \mu + z\sigma$

$x = 12.2 + (-1.34)(1.4)$

$= \boxed{10.324 \text{ oz.}}$



6. Suppose you take a random sample of n = 38 cats and find that the sample mean weight is 10.3 pounds with a sample standard deviation of 1.5 pounds. Build a 95% confidence interval for the population mean weight of cats.

$\bar{x} = 10.3$

$s = 1.5$

$n = 38$

σ not known

$c = 95\% = 0.95$

$E = t_c\left(\frac{s}{\sqrt{n}}\right)$

$t_c = 2.026$ (from chart)

$E = (2.026)\left(\frac{1.5}{\sqrt{38}}\right)$

$E = 0.493$

Confidence interval:

$10.3 - 0.493 < \mu < 10.3 + 0.493$

$\boxed{9.81 < \mu < 10.79}$

7. Suppose you take a random sample of n = 400 cats and find that 130 of the cats in the sample are grey. Build a 90% confidence interval for the population proportion of grey cats.

$\hat{p} = \frac{x}{n}$ $\hat{p} = \frac{130}{400} = 0.325$ $\hat{q} = 0.675$ $c = 0.90$ $z_c = 1.645$

$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.645\sqrt{\frac{(0.325)(0.675)}{400}}$

≈ 0.039

$\hat{p} - E < p < \hat{p} + E$

$0.325 - 0.039 < p < 0.325 + 0.039$

$0.286 < p < 0.364$

With 90% confidence we say that the population proportion of gray cats is between 0.286 and 0.364.

8. If you wish to estimate the population mean weight of cats to within 0.7 pounds with 99% confidence, what is the minimum sample size needed? From previous info, you know that the population standard deviation for cat weights is about 1.5 pounds.

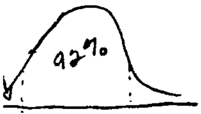
$E = 0.7$ lbs. $\sigma = 1.5$ lbs. $c = 0.99$ $z_c = 2.576$ $n = ?$

$n = \left(\frac{z_c\sigma}{E}\right)^2$

$n = \left(\frac{2.576 \cdot 1.5}{0.7}\right)^2$

$\approx 30.47 \Rightarrow 31$ cats

9. You wish to estimate the population proportion of cats that live in a house that also has a dog to within a two point margin of error, with 92% confidence. What is the cat sample size will you need?



$C = 0.92 + 0.04$
$= 0.96$
$z_c = 1.75$

$E = 0.02 \quad \hat{p} = 0.5 \quad \hat{q} = 0.5$

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$$

$$n = (0.5)(0.5)\left(\frac{1.75}{0.02}\right)^2$$

$$n = 1914.0625 \Rightarrow \boxed{1{,}915 \text{ cats}}$$

10. You take a random sample of five chihuahuas, and find that their weights in pounds are as follows: 5.3, 4.7, 7.2, 7.1, 7.4. Assuming that weights are normally distributed, build a 90% confidence interval for the population mean chihuahua weight.

$C = 0.90 \quad t_c = 2.132 \quad \bar{x} = 6.34 \quad s = 1.246 \quad \sigma$ not known
$df = 5 - 1 = 4$

$$E = t_c \cdot \frac{\sigma}{\sqrt{n}}$$
$$= \frac{2.132 \cdot 1.246}{\sqrt{5}}$$
$$\approx 1.188$$

$$\bar{x} - E < \mu < \bar{x} + E$$
$$6.34 - 1.188 < \mu < 6.34 + 1.188$$
$$\boxed{5.152 < \mu < 7.528}$$

11. Discuss how you would judge the validity of your answer to problem 10 if you did NOT know that weights were normally distributed.

You cannot build a valid confidence interval with 5 samples unless you know that the weights are normally distributed. You would need to have 30 or more samples.