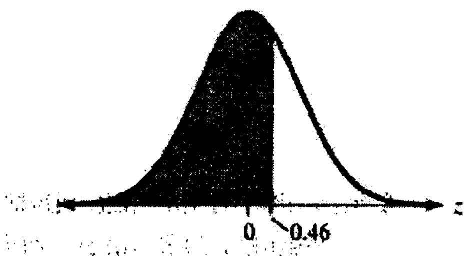
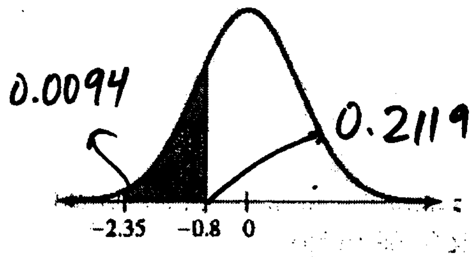
Illowsky – Chapt. 6, 7, & 8
Larson – Chapt. 5 & 6 B

Math 123, Fall 16, Midterm 3 Name: Solutions
Instructor: Saba Gerami **Total: 90 Points**

Directions:
- Show all your work. You only receive half of the points if you do not explain your reasoning.
- You can use a graphing calculator.
- You may not use cell phone, or notes.
- **Round to two decimal places.**

1. Find the area of the indicated region under the standard normal curve. *(4 points)*





a) 0.6772

b) 0.2119 − 0.0094 = 0.2025

2. *(4 points)*
a) Write two differences between a normal distribution and the standard normal distribution.

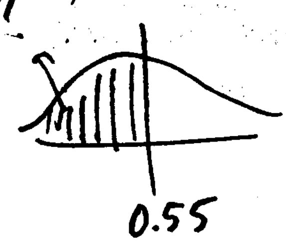
Their μ's are not necessarily the same.
Their σ's are not necessarily the same.

b) Write two similarities between a normal distribution and the standard normal distribution.

Both are bell-shaped.
Both are symmetric

3. Find the indicated area under the standard normal curve. *(6 points)*

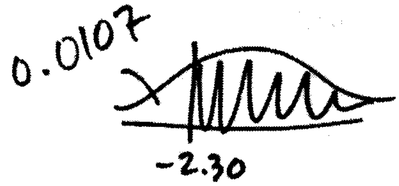
? a) To the left of $z = 0.55$



0.7088 1pt

9

b) To the right of $z = -2.30$



$1 - 0.0107$

$= 0.9893$

1

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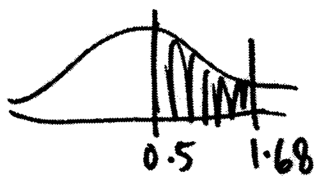
c) Between $z = 0.5$ and $z = 1.68$



area

$z = 0.5 \rightarrow 0.6915$

$z = 1.68 \rightarrow 0.9535$

$0.9535 - 0.6915 = 0.2620$

2

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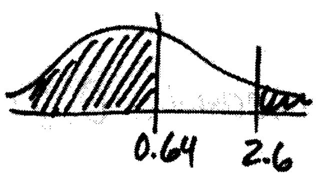
c) To the left of $z = 0.64$ or to the right of $z = 2.6$



on right

$z = 2.6 \longrightarrow$ area $= 0.9953 \rightarrow 1 - 0.9953$

$= 0.0047$

2

$z = 0.64 \rightarrow$ area $= 0.7389$

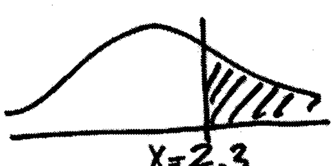
$\boxed{\text{area} = 0.7389 + 0.0047 = 0.7436}$

4. In a study of bumblebee bats, the weights were normally distributed with a mean 1.8 grams and a standard deviation of 0.33 gram. Find the probability that a randomly selected bat weighs:

$\mu = 1.8,\ \sigma = 0.33$

(6 points)

a) more than 2.3 grams.

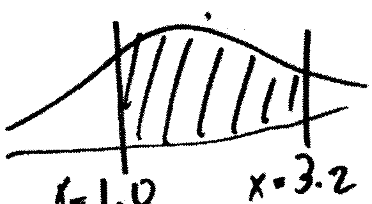


$z = \frac{X - \mu}{\sigma} = \frac{2.3 - 1.8}{0.33} \approx 1.52 \rightarrow$ left area $= 0.9357 \rightarrow$ right area $=$

$1 - 0.9357$

$= \boxed{0.0643}$

b) between 1.0 and 3.2 grams.



bw area $= 0.9998 - 0.0078$

$= \boxed{0.9920}$

$z = \frac{1 - 1.8}{0.33} = -2.42$

area

$= 0.0078$

$z = \frac{3.2 - 1.8}{.33} = 4.24$

area

$= 0.9998$

Bc it's the highest value on table

11

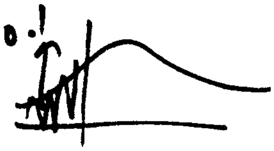
from left

5. Find the z-score that corresponds to the following cumulative area or percentile.

(4 points)

a) 0.1

0.1



$z = -1.28$

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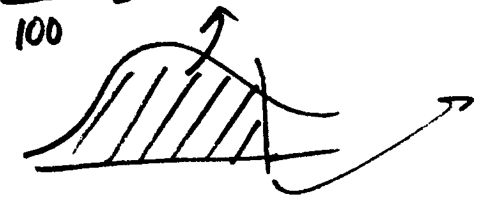
b) P_{95}

$\frac{95}{100} = 0.95$



$z = 1.645$

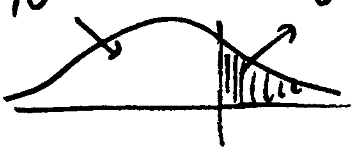
$\mu = 135$

$\sigma = 12$

6. Scores for Bar exam were normally distributed with a mean of 135 and standard deviation of 12 in 2016. An agency only hires applicants with scores in the top 10%. Bardo is graduating from a law school soon. What is the lowest score he can earn in order to be eligible to be hired by the agency?

(4 points)

0.90 $\qquad$ $0.10 = 10\%$



for 0.9 area $\rightarrow z = 1.28$

now, we find x for that z-score.

$X = \mu + z\sigma = 135 + (1.28)12 = \boxed{150.36}$

lowest score

7. The sample size n, probability of success p, and probability of failure q are given for a binomial experiment.

(6 points)

n=19 $\qquad$ p=0.23 $\qquad$ q=0.77

a) Can the normal distribution be used to approximate the random variable x? Explain your reasoning.

$np = 19(0.23) = 4.37 \not\geq 5$

2pts

$nq = 19(0.77) = 14.63$.

Not normal.

/10

$\mu = np = 4.37$

$\sigma = \sqrt{npq} = 1.83$

b) A binomial probability is given: $P(x = 116)$

Use a continuity correction to convert the binomial probability to a normal distribution probability.

$P(115.5 < X < 116.5)$

c) Find $P(x = 116)$.

$\boxed{\text{We cannot bc not normal}}$

Partial pts also given to students who did it:

$z = \frac{X - \mu}{\sigma} = \frac{116.5 - 4.37}{1.83} = 61.27$

$z = \frac{115.5 - 4.37}{1.83} = 60.73$ — not on table.

8. *The annual per capita waste of food from supermarkets is normally distributed with a mean of 185.2 pounds and standard deviation of 35.5 pounds. Random samples of 40 are drawn from this population, and the mean of each sample is determined.* *(6 points)*

$\mu = 185.2$

$\sigma = 35.5$

$n = 40$

a) Use the central Limit Theorem to find the mean and standard deviation of the following sampling distribution of sample means.

$$\mu_{\bar{x}} = \mu = 185.2$$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{35.5}{\sqrt{40}} = 5.61$$

b) What is the probability that the mean value of wasted food from supermarkets is more than 250 lbs annually per capita?

$$P(\bar{x} > 250) = P(z > 11.54) \approx 1 - 0.9998 \approx \boxed{0.0002}$$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{250 - 185.2}{5.61} \approx 11.54$$

highest value 0.9998

10

$P = 0.52$ $\quad \mu = np = 23.4$

$q = 0.48$ $\quad \sigma = \sqrt{npq} = 3.35$

$n = 45$

9. *A survey found that 52% of AHC students sells their book back to the bookstore. You randomly select 45 AHC students ages 18- 24 and ask them whether they do that or not.*

(6 points)

a) Use a continuity correction to find the probability that the number who sell their books backs is less than or equal to 30.

$$P(X \leq 30) = P(X < 30.5) = P(Z < 2.12) = \boxed{0.9830}$$

$$Z = \frac{X - \mu}{\sigma} = \frac{30.5 - 23.4}{3.35} \approx 2.12$$

b) Use a continuity correction to find the probability that the number who sell their books backs is greater than 40.

highest area is 0.9998

$$P(X > 40) = P(X > 40.5) = P(Z > 5.10)$$

$$= 1 - 0.9998 = 0.0002$$

10. Find the following values for the given confidence level C.

(4 points)

a) t_c and $-t_c$ when C = 90% and n=28.

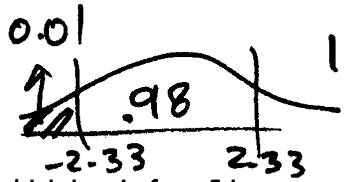
$df = 28 - 1 = 27 \rightarrow t_c = 1.703$

$-t_c = -1.703$

b) Z_c and $-Z_c$ when C= 98%

$-Z_c = -2.33$

$Z_c = 2.33$



$1 - 0.98 = 0.02$ twotails

one tail $\frac{0.02}{2} = 0.01$

11. For the same sample statistics, which level of confidence would produce the widest confidence interval? Choose the correct answer below. *(2 points)*

1. ~~90%~~, because as the level of confidence decreases, Z_c decreases.
2. ~~90%~~, because as the level of confidence decreases, Z_c increases.
3. 99%, because as the level of confidence increases, Z_c increases.
4. 99%, because as the level of confidence increases, Z_c decreases.

↑ level of confidence ⇔ ↑ %

/12

12. You construct a 95% confidence interval for a population mean using a random sample. The confidence interval is $24.9 < \mu < 31.5$
Is the probability that μ is in this interval 0.95? Choose the correct answer below. *(2 points)*

1. Yes. The probability that μ is in this interval is 0.95.
2. No. With 95% confidence, the mean is in the interval (24.9,31.5).
3. No. If a large number of samples are taken the probability that the mean is in the interval is 0.95.
4. No. Approximately 95% of the sampled data will fall in the confidence interval.

For problem 13- 15, number your steps as in your formula sheet to receive full credit.

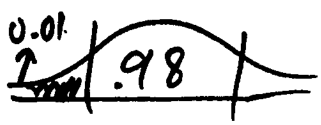
13. The mean room and board expenses per year at four-year colleges is \$15,400 among 100 students. Construct a 98% confidence interval for population mean. Assume room and board expenses have standard deviation of \$1300 based on recent studies. → sample so $\bar{X} = 15{,}400$ $n = 100$

not sample → $\sigma = 1300$ *(12 points)*

uideline (a)

(1). Verify sample random ✓
. $n = 100 \geq 30$ ✓

(2) $\bar{X} = 15{,}400$

(3) 

two tail $= 1 - 0.98 = 0.02$
one tail $= \frac{0.02}{2} = 0.01$

$-z_c = -2.33$ $z_c = 2.33$

(4) $E = z_c \cdot \frac{\sigma}{\sqrt{n}} = 2.33 \times \frac{1300}{\sqrt{100}} = 302.9$

(5) $15{,}400 - 302.9 < \mu < 15{,}400 + 302.9$
$15097.1 < \mu < 15702.9$

14

(6) I am 98% confident that μ falls bw 15097.1 & 15702.9

$n=35$ $\quad \hat{P} = \frac{27}{35}$, $\hat{q} = 1 - \frac{27}{35} = \frac{8}{35}$

14. In a survey of 35 students in Saba's statistics course, 27 said that they absolutely love listening to Disney music during worksheet hours! Construct a 95% confidence interval for the population proportions of students in all Saba's courses who absolutely love listening to Disney music during worksheet hours. *(12 points)*

Guideline (c)

1) $n=35$, $X=27$, $\hat{p} = \frac{27}{35}$, $\hat{q} = \frac{8}{35}$

2) $n\hat{p} = 35\left(\frac{27}{35}\right) = 27 \geq 5$ ✓ $\quad n\hat{q} = 35\left(\frac{8}{35}\right) = 8 \geq 5$ ✓

3) 0.025 $\quad$ 0.95 $\quad$ two tail $= 1 - 0.95 = 0.05$

one tail $= \frac{0.05}{2} = 0.025$

$-z_c = -1.96$ $\quad z_c = 1.96$

4) $E = z_c\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right) = 1.96\left(\sqrt{\frac{\frac{27}{35} \cdot \frac{8}{35}}{35}}\right) \approx 0.139$

5) $\frac{27}{35} - 0.139 < P < \frac{27}{35} + 0.139$

$0.63 < P < 0.91$

6) I am 95% confident that p falls b/w 0.63 & 0.91

15. You randomly select 16 coffees and measure their temperatures. You get an average of 162 Fahrenheit and a standard deviation of 10 Fahrenheit. Construct a 80% confidence interval for the population's mean of coffee's temperature. Assume the temperatures are normally distributed. *(12 points)*

$n=16$ → Sample
$\bar{x} = 162$
$s = 10$

Guideline (b)

1. Verify • Sample random ✓
   • Pop. normal ✓

2. $\bar{x} = 162$, $s = 10$

3. $df = 16 - 1 = 15$ $\quad$ .80 $\quad -t_c \quad t_c$ $\quad t_c = 1.341$, $-t_c = -1.341$

4. $E = t_c \cdot \frac{s}{\sqrt{n}} = 1.341 \frac{(10)}{\sqrt{16}} = 3.35$

5. $162 - 3.35 < \mu < 162 + 3.35$

$158.65 < \mu < 165.35$

6. I am 80% confident that μ falls b/w 158.65 & 165.35

$/24$