FUNCTION FACTS

Definition of a Function:

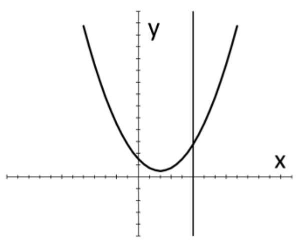
A function is a rule that describes how one quantity depends upon another.

- $f(x) = y$ is read "f of x."
- The output variable, y is the dependent variable because it depends on the input variable, x which is called the independent variable.
- For each input x, there is only one possible output y.

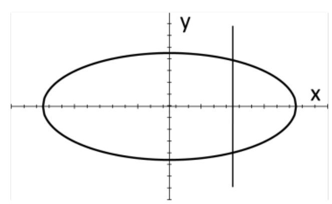
  Example: The set of points {(1,2), (2,4), (3,-1), (4,4)} is a function.

  The set of points {(1,2), (2,4), (3,-1), (3,4)} is not a function since an input of 3 yields more than one output.

Vertical Line Test: This tests whether or not a relation between two variables is a function. If a vertical line crosses the curve more than once, the relation is not a function.



This is a function.



This is **not** a function.

Domain: The domain is the set of all possible values of x for which the function $f(x)$ exists.

- x cannot cause a denominator to be zero.
- If x is under a square root (or any even root) sign, x cannot cause the expression under the root sign to be negative (when using real numbers).
- x must be greater than 0 for $y = \log_b x$.

Range: The range is the set of all possible values of the function, that is, the output variable, y.

Values of Functions:

Example: Let $f(x) = x^2 + 4x - 3$.

Find $f(2)$: $f(2) = 2^2 + 4(2) - 3 = 4 + 8 - 3 = 9$

Find $f(x + 1)$: $f(x + 1) = (x + 1)^2 + 4(x + 1) - 3 = x^2 + 6x + 2$

Algebra of Functions:

Sum: $(f + g)(x) = f(x) + g(x)$	Difference: $(f - g)(x) = f(x) - g(x)$
Product: $(fg)(x) = f(x) \cdot g(x)$	Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ for $g(x) \neq 0$

Examples of the algebra of functions: Let $f(x) = 2x$ and $g(x) = x - 1$.

Find $(f+g)(x)$: $f(x) + g(x) = (2x) + (x-1) = 3x - 1$

Find $\left(\frac{f}{g}\right)(x)$: $\left(\frac{f}{g}\right)(x) = \frac{2x}{x-1}$, $x \neq 1$

<u>Composite Functions:</u>

Composite functions are created when the input of one function is the output of another function.

- $(f \circ g)(x) = f(g(x))$ and is read "f of g of x."
- The domain of $(f \circ g)(x)$ is the set of all values of x such that:
  - x is in the domain of g and $g(x)$ is in the domain of f
- When working a problem:
  - Since the output of $g(x)$ is the input of the function f, work the inside parentheses first by substituting x into $g(x)$ and then use that solution as the input for the function f.
  - TIP: When substituting an expression or constant into an equation, always put parentheses ( ) around it.

Example 1: Let $f(x) = 2x$ and $g(x) = x - 1$. Then,

$$(f \circ g)(x) = f\big(g(x)\big) = f(x-1) = 2(x-1) = 2x - 2$$
$$(g \circ f)(x) = g\big(f(x)\big) = g(2x) = 2x - 1$$

Example 2: Let $f(x) = x^2 - x + 1$ and $g(x) = 3x$. Then,

$$(f \circ g)(x) = f\big(g(x)\big) = f(3x) = (3x)^2 - (3x) + 1 = 9x^2 - 3x + 1$$
$$(g \circ f)(x) = g\big(f(x)\big) = g(x^2 - x + 1) = 3(x^2 - x + 1) = 3x^2 - 3x + 3$$

<u>One-To-One Functions and the Horizontal Line Test:</u>

One-To-One means that for each output y, there is only one possible x input. If a horizontal line crosses a curve <u>more than once</u>, it is not a one-to-one function.

Note:

- Only one-to-one functions have inverse functions.
- All linear functions are one-to-one functions, except when the slope = 0.

<u>Inverse Functions:</u>

An inverse function, f^{-1}, "undoes" the action of a function so that $f^{-1}\big(f(x)\big) = x$ and $f(f^{-1}(x))$.

Example: To find the inverse of $f(x) = x + 2$ follow these four steps:

1. Replace $f(x)$ with y: $y = x + 2$
2. Switch x and y: $x = y + 2$
3. Solve for y: $y = x - 2$
4. Replace y with $f^{-1}(x)$: $f^{-1}(x) = x - 2$