CALCULUS 181
Stewart – Chapter 3

Name: Key

Chapter 3 Exam

Differentiate. Show all your work.

1. $y = \frac{\sqrt{x}}{2+x}$

$$\frac{dy}{dx} = \frac{(2+x)\left(\frac{1}{2}x^{-1/2}\right) - \sqrt{x}\,(1)}{(2+x)^2}$$

$$= \frac{x^{-1/2}\left(1 + \frac{1}{2}x - x\right)}{(2+x)^2}$$

$$= \frac{x^{1/2}\left(1 - \frac{1}{2}x\right)}{(2+x)^2}$$

2. $y = \sec\theta\tan\theta$

$$\frac{dy}{dx} = (\sec\theta)(\sec^2\theta) + (\tan\theta\sec\theta)(\tan\theta)$$

$$= \sec^3\theta + \tan^2\theta\sec\theta$$

3. $f(x) = (2x-3)^4(x^2+x+1)^5$

$$f'(x) = 4(2x-3)^3(2x)(x^2+x+1)^5 + (2x-3)^4\left(5(x^2+x+1)^4(2x+1)\right)$$

$$= (2x-3)^3(x^2+x+1)^4\left(4(2x)(x^2+x+1) + (2x-3)(2x+1)\right)$$

$$= (2x-3)^3(x^2+x+1)^4\left(8x^3 + 8x^2 + 8x + 4x^2 - 4x - 3\right)$$

$$= (2x-3)^3(x^2+x+1)^4\left(8x^3 + 12x^2 + 4x - 3\right)$$

4. $y = \cot^2(\sin\theta)$

$$\frac{dy}{d\theta} = 2\cot(\sin\theta)\left(\cot(\sin\theta)\right)'$$

$$= 2\cot(\sin\theta)\left(-\csc^2(\sin\theta)\right)(\sin\theta)'$$

$$= 2\cot(\sin\theta)\left(-\csc^2(\sin\theta)\right)(\cos\theta)$$

$$= -2\cot(\sin\theta)\csc^2(\sin\theta)\cos\theta$$

5. Find an equation of the tangent line to the curve $y = \frac{2}{(1+e^{-x})}$ at the point $(0, 1)$

$$\frac{dy}{dx} = \frac{(1+e^{-x})(0) - (2)(-e^{-x})}{(1+e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2}$$ plug in 0 for x.

$$m = \frac{2}{2^2}$$

$$m = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x)$$

$$y = \frac{1}{2}x + 1$$

6. Find $\frac{dy}{dx}$ by implicit differentiation.

$\cos(xy) = 1 + \sin y$

$$-\sin(xy)\left(x\frac{dy}{dx} + y\right) = \cos y\frac{dy}{dx}$$

$$-x\frac{dy}{dx}\sin(xy) - y\sin(xy) = \cos y\frac{dy}{dx}$$

$$-y\sin(xy) = \cos y\frac{dy}{dx} + x\frac{dy}{dx}\sin(xy)$$

$$-y\sin(xy) = \frac{dy}{dx}\left(\cos y + x\sin(xy)\right)$$

$$\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos y + x\sin(xy)}$$

7. Use logarithmic differentiation to find the derivative of the function.

$y = x^{\cos x}$

$\ln y = \cos x \ln x$

$\frac{1}{y}\frac{dy}{dx} = (\cos x)\left(\frac{1}{x}\right) + (\ln x)(-\sin x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{x} - \ln x \sin x$

$\frac{dy}{dx} = y\left(\frac{\cos x}{x} - \ln x \sin x\right)$

$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right)$

8. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$. When does the ball reach it's maximum height?

$\frac{ds}{dt} = 80 - 32t$

$0 = 80 - 32t$

$\frac{5}{2} = t$

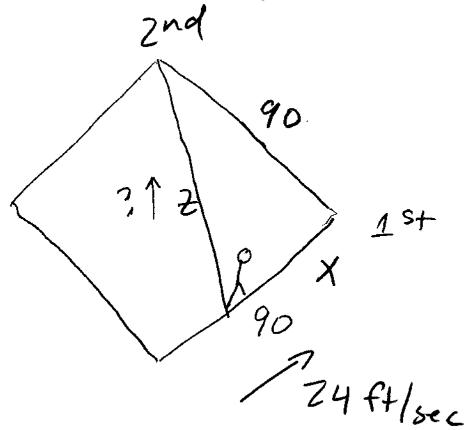
$+ \qquad -$

$\frac{5}{2}$

max

$t = \frac{5}{2}$ seconds

9. A baseball diamond is a square with sides 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base? Please draw a picture and label your variables on the picture.



$$\frac{dx}{dt} = -24\,\text{ft/sec}$$

$$\frac{dz}{dt} = ?$$

$$x = 45$$

$$x^2 + 90^2 = z^2$$

$$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$$

$$45^2 + 90^2 = z^2$$

$$100.6 = z$$

$$2(45)(-24) = 2(100.6)\frac{dz}{dt}$$

$$-10.7 = \frac{dz}{dt}$$

The distance to second base decreases at a rate of 10.7 ft/sec.