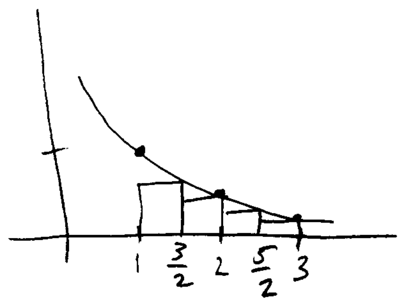
CALCULUS 181
Stewart – Chapter 5

Name: Key

Chapter 5 Exam

1. Estimate the area under the graph of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 3$ using four approximating rectangle and right endpoints. Sketch a picture.



$A = \frac{1}{2}\left(\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right)$

$A = \frac{19}{20}$

Evaluate each integral

2. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (sec^2 x)\, dx$

$$\int_{\pi/4}^{\pi/3} \sec^2 x \, dx = \tan x \Big|_{\pi/4}^{\pi/3}$$

$$= \tan \pi/3 - \tan \pi/4$$

$$= \sqrt{3} - 1$$

3. $\int_{-1}^{2} (3x-2)(x+1)\, dx$

$$\int_{-1}^{2} (3x^2 + x - 2)\, dx = x^3 + \frac{1}{2}x^2 - 2x \Big|_{-1}^{2}$$

$$= (8 + 2 - 4) - (-1 + \tfrac{1}{2} + 2)$$

$$= 6 - \frac{3}{2}$$

$$= \frac{9}{2}$$

4. $\int_1^2 \frac{x^3+3x^6}{x^4}\,dx$

$$\int_1^2 \frac{x^3+3x^6}{x^4}\,dx = \int_1^2 \left(\frac{1}{x} + 3x^2\right) dx$$

$$= \ln x + x^3 \Big|_1^2$$

$$= \ln 2 + 8 - (\ln 1 + 1)$$

$$= \ln 2 + 7$$

5. $\int (x^2 + 1 + \frac{1}{x^2+1})\,dx$

$$\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \frac{1}{3}x^3 + x + \tan^{-1} x + C$$

6. $\int \sin x \sqrt{1+\cos x}\,dx$

Let $u = 1+\cos x$

$$\frac{du}{dx} = -\sin x$$

$$-du = \sin x\,dx$$

$$\int \left(\sin x \sqrt{1+\cos x}\right) dx = -\int \sqrt{u}\,du$$

$$= -\frac{2}{3}u^{3/2} + C$$

$$= -\frac{2}{3}(1+\cos x)^{3/2} + C$$

7. $\int_1^e \frac{\ln x}{x}\,dx$

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

if $x = e$, $u = \ln e$
$= 1$

if $x = 1$, $u = \ln 1$
$= 0$

$$\int_0^1 u\,du = \frac{1}{2}u^2 \Big|_0^1$$

$$= \frac{1}{2}$$

8. $\int \sin x \cos(\cos x)\,dx$

Let $u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$-\int (\cos u)\,du = -\sin u + c$$

$$= -\sin(\cos x) + c$$

9. $\int \frac{\sec\theta \tan\theta}{1 + \sec\theta}\,d\theta$

Let $u = 1 + \sec\theta$

$du = \sec\theta \tan\theta\,d\theta$

$$\int \frac{1}{u}\,du = \ln u + c$$

$$= \ln(1 + \sec\theta) + c$$

10. If $f'(x) = 1 + 3\sqrt{x}$, find $f(x)$ if $f(4) = 25$

$$f'(x) = 1 + 3x^{1/2}$$

$$f(x) = x + 3\left(\frac{2}{3}x^{3/2}\right) + c$$

$$f(x) = x + 2x^{3/2} + c$$

$$25 = 4 + 2(4)^{3/2} + c$$

$$25 = 4 + 16 + c$$

$$5 = c$$

$$f(x) = x + 2x^{3/2} + 5$$