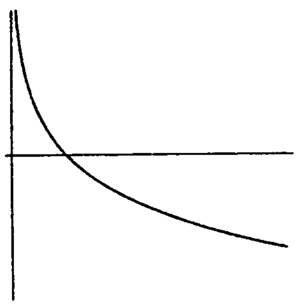
Math 181
Calculus 1

Name ______________________

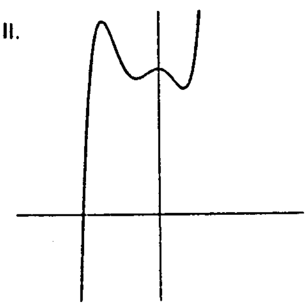
TEST 1

1. Match the following graphs with the formulas.

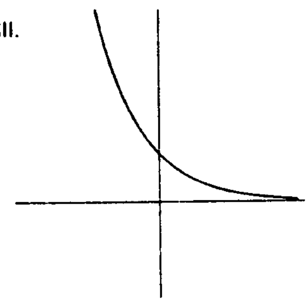
I.



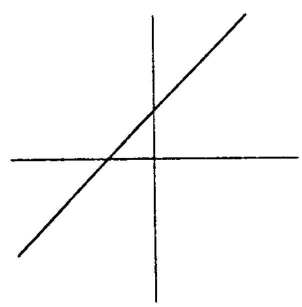
II.



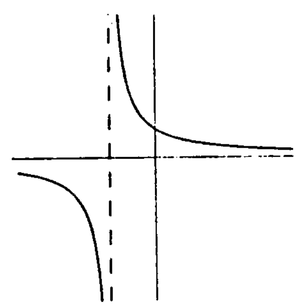
III.



IV.



V.



(a) $\ln(e^x) + 1$
(b) $-2\ln x$
(c) e^{-x}
(d) $x^5 + 2x^4 - x^3 - 2x^2 + 5$
(e) $\dfrac{1}{x+1}$

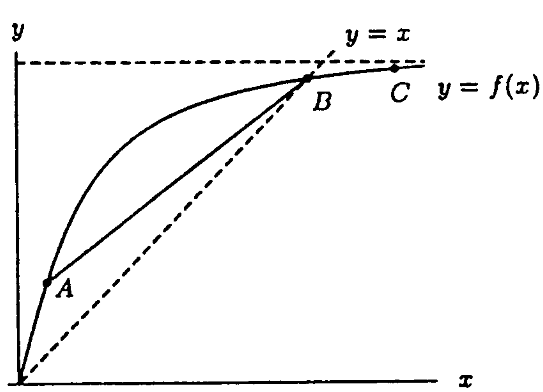
2. Give expressions for $f(x), g(x), h(x)$ which agree with the following table of values.

x	$f(x)$	$g(x)$	$h(x)$
0	−7	0	–
1	−4	2	5
2	−1	8	2.50
3	2	18	1.66...
4	5	32	1.25
5	8	50	1

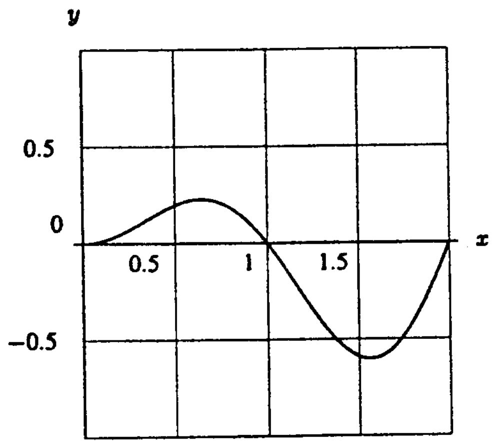
3. You are offered two jobs starting on July 1st of 1994. Firm A offers you \$40,000 a year to start and you can expect an annual raise of 4% every July 1st. At firm B you would start at \$30,000 but can expect an annual 6% increase every July 1st. On July 1st of which year would the job at firm B first pay more than the job at firm A?

4. If the graph of $y = f(x)$ is shown below, arrange in ascending order (i.e., smallest first, largest last):

$f'(A)$ $f'(B)$ $f'(C)$ slope AB the number 1 the number 0



5. Below is the graph of a function f. Sketch the graph of its derivative f' on the same axes.

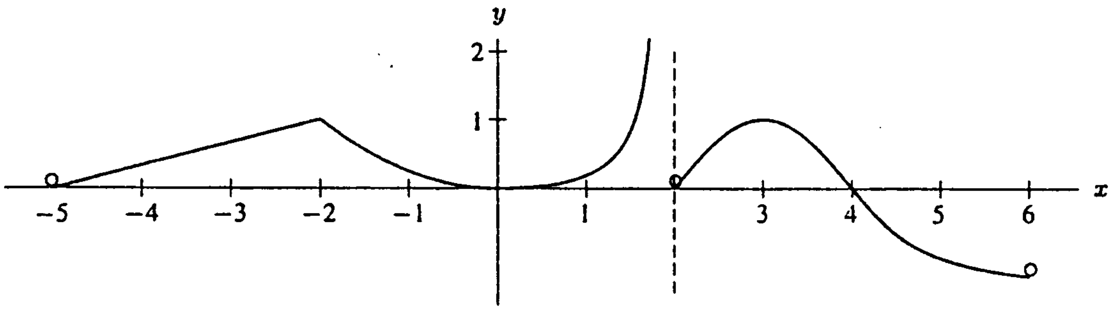


6. Given the following data about a function f,

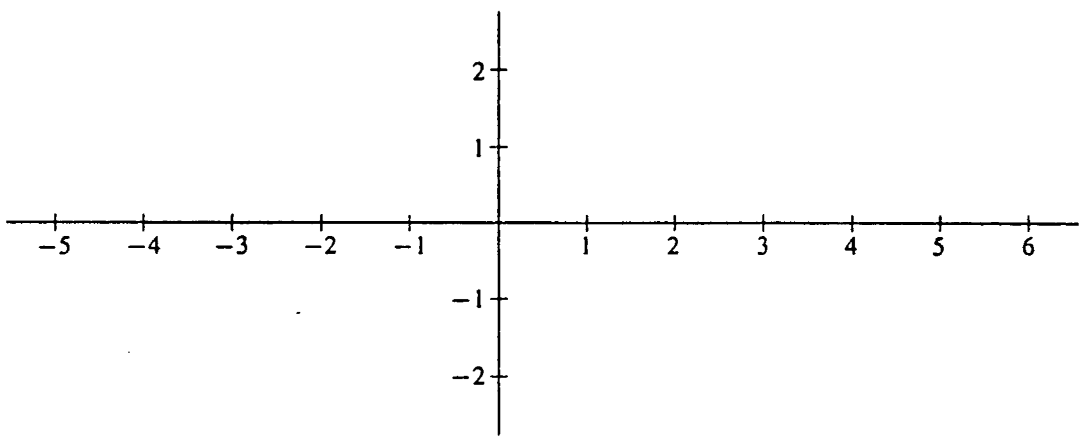
x	3.0	3.2	3.4	3.6	3.8
$f(x)$	8.2	9.5	10.5	11.0	13.2

(a) Estimate $f'(3.2)$ and $f'(3.5)$.
(b) Give the average rate of change of f between $x = 3.0$ and $x = 3.8$.
(c) Give the equation of the tangent line at $x = 3.2$.

7. Consider the function $y = f(x)$ graphed below. (Notice that $f(x)$ is defined for $-5 < x < 6$, *except* $x = 2$.)



(a) For what values of x (in the domain of f) is $f'(x) = 0$?
(b) For what values of x (in the domain of f) is $f'(x)$ positive?
(c) For what values of x (in the domain of f) is $f'(x)$ negative?
(d) For what values of x (in the domain of f) is $f'(x)$ undefined?
(e) Based on your answers to the above questions, make a sketch of $y = f'(x)$ on the axes below. Make your sketch as precise as possible.

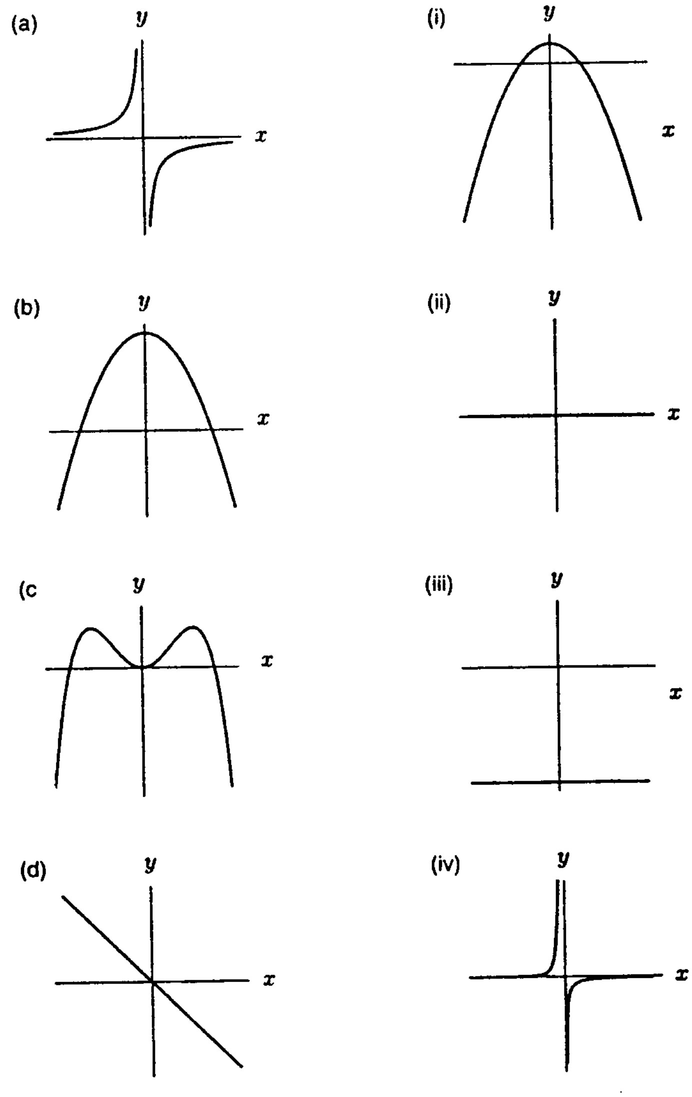


8. Suppose that $f(T)$ is the cost to heat my house, in dollars per day, when the outside temperature is T degrees Fahrenheit.

(a) What does $f'(23) = -0.17$ mean?
(b) If $f(23) = 7.54$ and $f'(23) = -0.17$, approximately what is the cost to heat my house when the outside temperature is 20°F?

9. Each graph in the right-hand column below represents the *second* derivative of some function shown in the left-hand column. Match the functions and their second derivatives.

Functions | Second Derivatives



Function (a) has second derivative

Function (b) has second derivative

Function (c) has second derivative

Function (d) has second derivative

10. The cost of mining a ton of coal is rising faster every year. Suppose $C(t)$ is the cost of mining a ton of coal at time t.

(a) Which of the following must be positive? (Circle those which are.)

(i) $C(t)$

(ii) $C'(t)$

(iii) $C''(t)$

(b) Which of the following must be increasing? (Circle those which are.)

(i) $C(t)$

(ii) $C'(t)$

(iii) $C''(t)$

(c) Which of the following must be concave up? (Circle those which are.)

(i) $C(t)$

(ii) $C'(t)$

(iii) $C''(t)$

EXTRA CREDIT

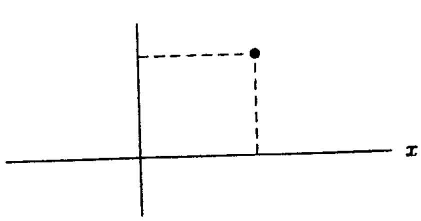
11. Given the following data about a function, f,

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

(a) Estimate $f'(4.25)$ and $f'(4.75)$.
(b) Estimate the rate of change of f' at $x = 4.5$.
(c) Find, approximately, an equation of the tangent line at $x = 4.5$.
(d) Use the tangent line to estimate $f(4.75)$.
(e) Estimate the derivative of f^{-1} at 2.

12 On the axes below, sketch a smooth, continuous curve (i.e., no sharp corners, no breaks) which passes through the point $P(3,4)$, and which clearly satisfies the following conditions:

- Concave up to the left of P
- Concave down to the right of P
- Increasing for $x > 0$
- Decreasing for $x < 0$
- Does *not* pass through the origin.



Math 181
Calculus 1

Name ______________________

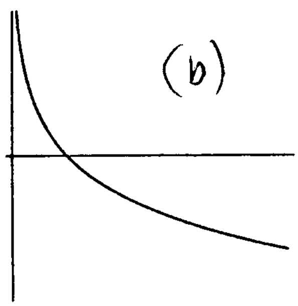
TEST 1

53 +10 Ex.C.

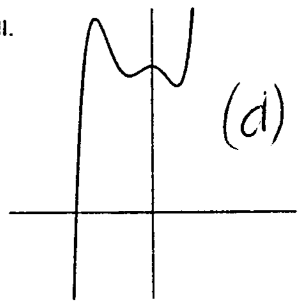
1. Match the following graphs with the formulas.

5

I.



II.



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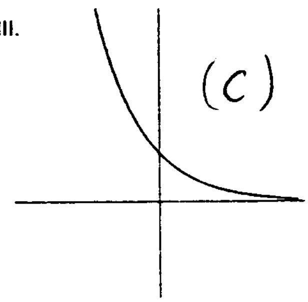
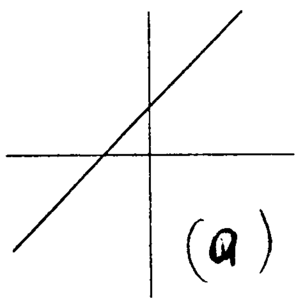


IV.



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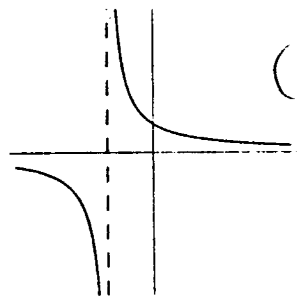


IV (a) $\ln(e^x) + 1$
I (b) $-2\ln x$
III (c) e^{-x}
II (d) $x^5 + 2x^4 - x^3 - 2x^2 + 5$
V (e) $\frac{1}{x+1}$

2. Give expressions for $f(x), g(x), h(x)$ which agree with the following table of values.

6

x	$f(x)$	$g(x)$	$h(x)$
0	−7	0	−
1	−4	2	5
2	−1	8	2.50
3	2	18	1.66...
4	5	32	1.25
5	8	50	1

$f(x) = 3x - 7$

$g(x) = 2x^2$

$h(x) = \frac{5}{x}$

3. You are offered two jobs starting on July 1st of 1994. Firm A offers you \$40,000 a year to start and you can expect an annual raise of 4% every July 1st. At firm B you would start at \$30,000 but can expect an annual 6% increase every July 1st. On July 1st of which year would the job at firm B first pay more than the job at firm A?

6

$$P_A = 40{,}000(1.04)^t \quad , \quad P_B = 30{,}000(1.06)^t$$

$$\cancel{A \pm B} \qquad P_A = P_B$$

$$40{,}000(1.04)^t = 30{,}000(1.06)^t$$

$$\left(\frac{1.04}{1.06}\right)^t = \frac{3}{4}$$

$$t \cdot \ln\left(\frac{104}{106}\right) = \ln\left(\frac{3}{4}\right)$$

$$t \approx 15.1$$

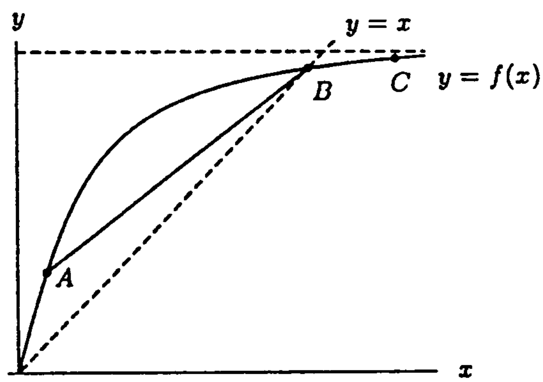
$$1994 + 16 = 2010$$

Ans. On July 1st 2010 the job at firm B would first pay more than the job at firm A.

4. If the graph of $y = f(x)$ is shown below, arrange in ascending order (i.e., smallest first, largest last):

5

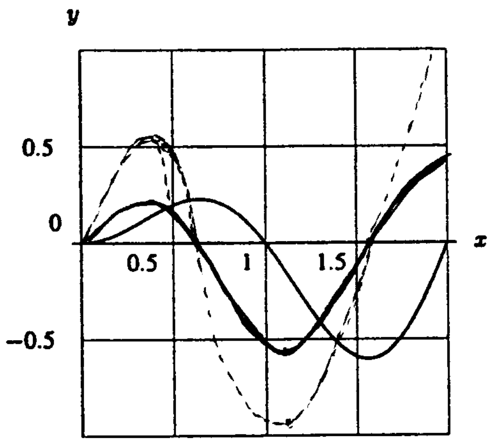
$f'(A)$ $\quad f'(B)$ $\quad f'(C)$ $\quad$ slope AB $\quad$ the number 1 $\quad$ the number 0



$$0 < f'(C) < f'(B) < \text{slope } AB < 1 < f'(A)$$

5. Below is the graph of a function f. Sketch the graph of its derivative f' on the same axes.

5



6. Given the following data about a function f,

6

x	3.0	3.2	3.4	3.6	3.8
$f(x)$	8.2	9.5	10.5	11.0	13.2

(a) Estimate $f'(3.2)$ and $f'(3.5)$.
(b) Give the average rate of change of f between $x = 3.0$ and $x = 3.8$.
(c) Give the equation of the tangent line at $x = 3.2$.

(a) $f'(3.2) \approx \frac{10.5 - 8.2}{0.4} = \frac{2.3}{0.4} = 5.75$

$f'(3.5) \approx \frac{11 - 10.5}{0.2} = \frac{0.5}{0.2} = 2.5$

(b) $\frac{13.2 - 8.2}{0.8} = \frac{5}{0.8} = 6.25$

(c) $m = f'(3.2)$, point $(3.2, 9.5)$

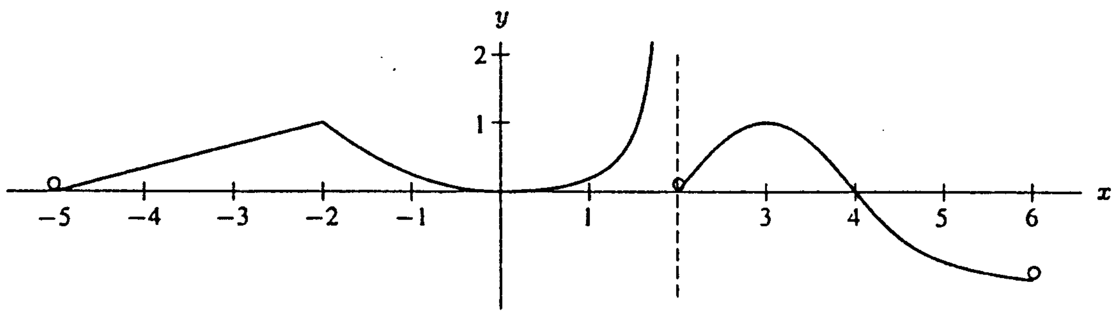
$y - y_1 = m(x - x_1)$

$y - 9.5 = 5.75(x - 3.2)$

$\boxed{y = 5.75x - 8.9}$

7. Consider the function $y = f(x)$ graphed below. (Notice that $f(x)$ is defined for $-5 < x < 6$, *except* $x = 2$.)

6



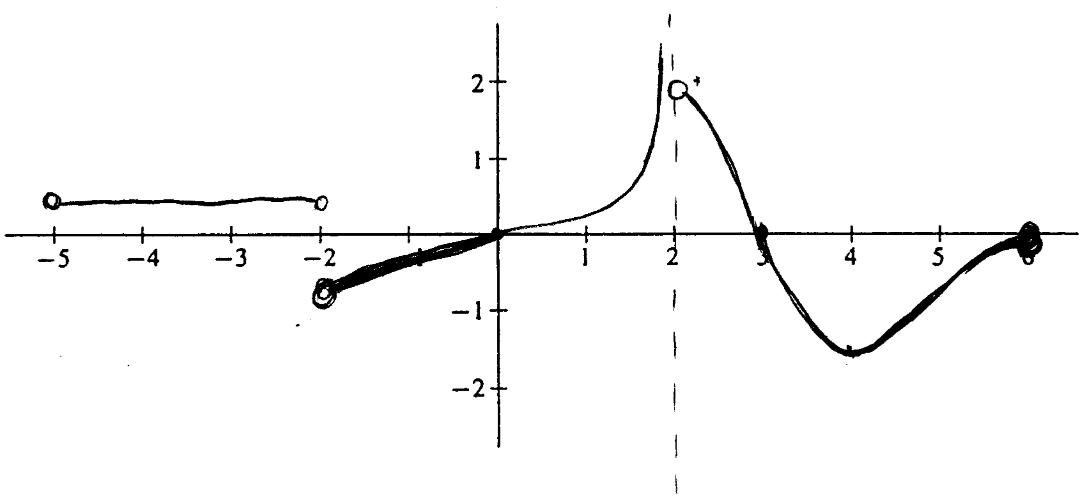
(a) For what values of x (in the domain of f) is $f'(x) = 0$? $x = 0$, $x = 3$

(b) For what values of x (in the domain of f) is $f'(x)$ positive? $-5 < x < -2$, $0 < x < 2$, $2 < x < 3$

(c) For what values of x (in the domain of f) is $f'(x)$ negative? $(-2, 0)$, $(3, 6)$

(d) For what values of x (in the domain of f) is $f'(x)$ undefined? $x = -2$,

(e) Based on your answers to the above questions, make a sketch of $y = f'(x)$ on the axes below. Make your sketch as precise as possible.



$f'(23) = -.17$ means that when the temperature outside is 23°F, the cost of heating the house will decrease by a rate of approximately 17¢ per day for each degree above 23.

8. Suppose that $f(T)$ is the cost to heat my house, in dollars per day, when the outside temperature is T degrees Fahrenheit.

4

(a) What does $f'(23) = -0.17$ mean?

(b) If $f(23) = 7.54$ and $f'(23) = -0.17$, approximately what is the cost to heat my house when the outside temperature is 20°F?

(a) If the outside temperature is 23°F, the cost to heat my house decreases 17¢, when the temperature increases 1°F.

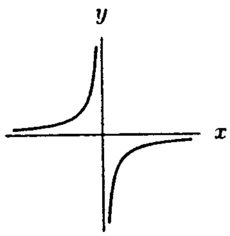
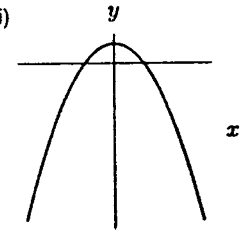
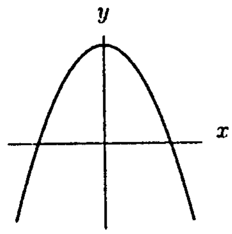
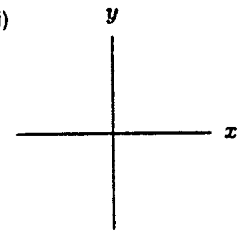
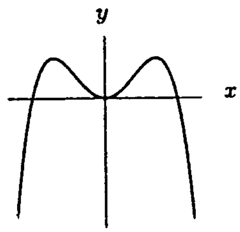
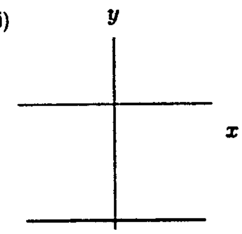
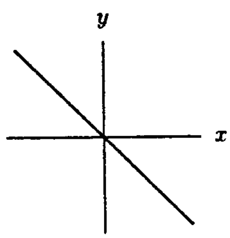
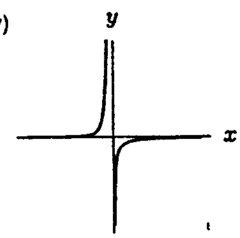
(b) $3 \times .17 = .51$

$7.54 + .51 = 8.05$

The cost will be approximately \$ 8.05.

9. Each graph in the right-hand column below represents the *second* derivative of some function shown in the left-hand column. Match the functions and their second derivatives.

4

Functions	Second Derivatives
(a) 	(i)  c
(b) 	(ii)  d
(c 	(iii)  b
(d) 	(iv)  a

Function (a) has second derivative iv

Function (b) has second derivative iii

Function (c) has second derivative i

Function (d) has second derivative ii

10. The cost of mining a ton of coal is rising faster every year. Suppose $C(t)$ is the cost of mining a ton of coal at time t.

6

(a) Which of the following must be positive? (Circle those which are.)
- (i) $C(t)$
- (ii) $C'(t)$
- (iii) $C''(t)$

(b) Which of the following must be increasing? (Circle those which are.)
- (i) $C(t)$
- (ii) $C'(t)$
- (iii) $C''(t)$

(c) Which of the following must be concave up? (Circle those which are.)
- (i) $C(t)$
- (ii) $C'(t)$
- (iii) $C''(t)$

11. Given the following data about a function, f,

5

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	10	8	7	4	2	0	-1

(a) Estimate $f'(4.25)$ and $f'(4.75)$.
(b) Estimate the rate of change of f' at $x = 4.5$.
(c) Find, approximately, an equation of the tangent line at $x = 4.5$.
(d) Use the tangent line to estimate $f(4.75)$.
(e) Estimate the derivative of f^{-1} at 2.

$$f''(4.5) \approx \frac{f'(4.75) - f'(4.5)}{4.75 - 4.5}$$

(a) $f'(4.25) \approx \frac{4-7}{4.5-4} = \frac{-3}{.5} = \boxed{-6}$

$f'(4.75) \approx \frac{2-4}{0.5} = \frac{-2}{.5} = \boxed{-4}$

(b) $f''(4.5) \approx \cancel{\frac{-4+(-6)}{2}} = \boxed{\cancel{-5}}$ $\qquad \frac{f'(4.75) - f'(4.25)}{0.5} = \frac{-4+6}{0.5} = \boxed{4}$

(c) $m = f'(4.5) = -5$, point $(4.5, 4)$

$y - 4 = -5(x - 4.5)$

$\boxed{y = -5x + 26.5}$

(d) $f(4.75) \approx f(4.5) + f'(4.5)(4.75 - 4.5)$

$f(4.75) \approx 4 + (-5)(0.25)$ $\qquad f(4.75) \approx -5 \cdot 4.75 + 26.5$

$\boxed{f(4.75) \approx 2.75}$

(e) $(f^{-1})'(2) \approx \frac{4.5 - 5.5}{4 - 0} = \frac{-1}{4} = \boxed{-0.25}$

12. On the axes below, sketch a smooth, continuous curve (i.e., no sharp corners, no breaks) which passes through the point $P(3, 4)$, and which clearly satisfies the following conditions:

5

- Concave up to the left of P
- Concave down to the right of P
- Increasing for $x > 0$
- Decreasing for $x < 0$
- Does *not* pass through the origin.